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Monetary Policy and Government Credit Programs

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Credit rationing is a common feature of most developing economies. In response to it, the governments of these countries often operate a number of programs intended to expand the supply of credit to the private sector. Expansionary monetary policy is often seen as a way of reducing the extent of credit rationing. We examine the consequences of a common policy tool in these economies: the use of expansionary monetary policy combined with direct central bank lending to inject credit. In the context of a small open economy we show that such a policy increases long-run production if and only if the economy is in a development trap. Moreover government credit programs often lead to endogenously arising aggregate volatility. Thus the case for government intervention in credit markets relies largely on the notion that output is artificially low because the economy is in a development trap. However, it is the case that the kind of policy we consider can be used to eliminate certain indeterminacies of equilibrium created by endogenous credit market frictions. *Journal of Economic Literature* Classification Numbers: E44, O16, O42. © 2002 Elsevier Science (USA)

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1. INTRODUCTION

It is often asserted that the presence of credit rationing gives monetary policy considerable scope to affect real economic activity.[1] In particular, the claim is that

[1] See, for instance, Blinder and Stiglitz (1983) or Stiglitz and Weiss (1992). More generally, this is the point of view implicit in the literature on the "credit channel" view of monetary policy.

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when credit is rationed, an "easier" monetary policy can expand the availability of credit. As a result, there can be a corresponding expansion of lending, investment, and production.

It is also the case that the rationing of credit seems to be especially prevalent in developing countries. Moreover, in developing country contexts, central banks are often actively involved in the allocation of credit. Indeed, the central bank often lends directly to banks engaged in making loans to "strategic" sectors of the economy or rediscounts a variety of private sector loans. Here, then, the connection between monetary expansion and the supply of credit is particularly tight: money growth is used to finance direct or indirect lending by the central bank.[2] If the claim about the link between monetary policy and credit availability in the presence of credit rationing is correct, this link should be particularly strong in developing countries.

However, expansionary monetary policy has other consequences in addition to its impact on the availability of credit. Most specifically, increases in the rate of money creation also increase the rate of inflation. And, it is empirically well-established that increases in the rate of inflation can have strong adverse consequences for credit market conditions[3] and consequently output growth.

These observations suggest an obvious question: which of these effects is likely to dominate? Or, alternatively, when (if at all) should a central bank use expansionary monetary policy as a means of injecting credit into an economy? Clearly, expansionary monetary policies can lead to an expansion of credit only if the consequences of credit injection are strong enough to offset the contractionary effects of higher rates of inflation on bank lending.

All of the issues mentioned thus far can be posed with reference to long-run equilibria. In addition, there is the question of how the use of monetary policy to inject credit affects equilibrium dynamics in a small open economy—a category which presumably includes most developing countries. In this regard, two facts are well established but not theoretically explained: (a) high average rates of inflation are strongly associated with high inflation variability, and (b) when the rate of inflation exceeds some critical level, further increases in inflation can lead to so-called *inflation crises* with dramatic consequences for output.[4] One of our goals is to provide theoretical explanations for these observations. We would also like to understand whether the use of monetary policy to fund credit extension can affect the determinacy of equilibrium.

In order to investigate these issues, we construct a conventional monetary growth model (Diamond, 1965), which has been reformulated so that capital investment requires access to external finance. In our model, credit rationing arises endogenously

[2] For example, in Turkey central bank rediscounting was the dominant instrument of monetary policy into the late 1980s ("The Turkish Economy and the Financial Sector," 1995, p. 12). Central bank rediscounting in Mexico is discussed, for example, by Chavez (1983).

[3] On this point see Boyd *et al.* (2000), Gonzalez-Vega (1984), Johnson (1983), or Khan *et al.* (2001).

[4] "Inflation crises" are discussed by Bruno and Easterly (1998).

because of a costly state verification (CSV) problem[5] in credit markets. Motivated by the presence of this rationing, the government (central bank) utilizes seigniorage income to rediscount bank lending or to subsidize private credit extension. Also, in keeping with the circumstances relevant to most developing countries, we focus on a small open economy. Furthermore, in our model, lenders are subjected to a reserve requirement, just as intermediaries have been for long periods of time in most developing economies.

In this framework, we are able to obtain the following results. First, if there is any steady state equilibrium where credit is rationed domestically, then there are typically at least two such equilibria. One has relatively high capital stock and output level; we refer to this as the high activity steady state. The other has a relatively low capital stock and output level. Moreover, for a wide range of parameter values both steady states can be approached. Hence development traps are possible; here these are the consequence of a CSV problem in credit markets.

The long-run aggregate consequences of using expansionary monetary policy to finance the direct extension of credit by the government turn out to depend very heavily on which of the steady state equilibria prevails. Under certain methods for financing central bank lending, we show that at the high activity steady state, an increase in the volume of government lending (rediscounting) necessarily reduces the long-run capital stock and output level, as well as the volume of private credit extended to domestic residents. Thus, government credit market interventions are counterproductive—at least with respect to long-run output—in the high activity state. This effect is reversed, however, whenever the economy is caught in a development trap. Only there can the expansion of credit associated with monetized government lending outweigh the negative implications of higher rates of inflation. In essence, then, the case for subsidized government lending as a means of stimulating production may make sense only if the economy is stranded in a low activity equilibrium to begin with.

With respect to shorter-run phenomena, we also show that the magnitude of government lending substantially affects the properties of dynamical equilibria, the scope for indeterminacy of equilibrium, and the potential for endogenously arising volatility. More specifically, when government credit programs are not too large, we show that the low activity steady state is a saddle, and the dynamical equilibrium paths approaching it do so monotonically. Matters are much different near the high activity steady state, however. In particular, we state conditions where extremely small government lending leads the high activity steady state to be a sink. Since our economy has only a single initial condition, the result is an indeterminacy; there are multiple equilibrium paths that approach the high activity steady state. Moreover, as the size of the government credit program increases, we state conditions under which dynamical equilibria approaching the high activity steady state must exhibit fluctuations. In effect, then, government lending programs become responsible for

[5] See Townsend (1979), Diamond (1984), Gale and Hellwig (1985), or Williamson (1986, 1987). The specific formulation we utilize is similar to that in Boyd and Smith (1997, 1998).

endogenously arising volatility. This volatility will be reflected in all endogenous variables, including the rate of inflation. Thus high average rates of inflation are associated with inflation variability, as we observe. Moreover, the potential of government credit extension financed by high enough rates of money creation to produce volatility, coupled with some of the findings described above, supports the view that these programs may interfere with macroeconomic performance.

However, our most important result is that the case for using monetary expansion to fund government lending depends on the economy being "trapped" in or near a low activity steady state. At or near the high activity steady state, monetary expansions—even if they are used to directly inject credit into the economy—adversely affect long-run capital formation and production. This finding is of interest in view of the fact that government credit interventions have widely been viewed as counterproductive.[6]

Interestingly, as government credit programs become very large, the properties of the dynamical equilibria near each steady state become very different. For large government interventions the low activity steady state can become a sink, while the high activity steady state becomes a saddle. When this is the case, the high activity steady state is no longer indeterminate. But, as we show, it is also the case that paths approaching the high activity steady state necessarily display endogenously arising fluctuations as they do so.

This last point deserves some emphasis. As a whole, our results imply that small government credit programs will be most conducive to large output levels, near the high activity steady state. But they can also easily lead to the indeterminacy of equilibrium. Large government programs have adverse consequences for long-run output (again, near the high activity steady state), and they can lead to endogenously generated volatility. But they are consistent with a determinate equilibrium. Thus here as elsewhere,[7] a tension between the determinacy and the "efficiency" of equilibrium can easily arise.

Intuitively speaking, what accounts for the steady state results we have described above? The answer has to do with the interaction between the CSV problem, the domestic reserve requirement, the operation of international capital markets, and the way the government lending program is financed. In a small open economy, agents lending domestically must receive the prevailing world real return on their investments. In addition, the domestic reserve requirement forces such agents to hold a portfolio consisting of domestic loans and domestic real balances. It is the return on this portfolio that must match the world rate of interest.

For a given domestic rate of inflation, this return is clearly determined by the rate of return on loans. In the presence of the CSV problem, the return on domestic loans depends on two factors: the domestic marginal product of capital, and the

[6] See Khatkhate (1982b), Bhatt (1988), Smith and Stutzer (1989), or Gale (1990) for a discussion of the notion that government interventions in credit markets have often been counterproductive.

[7] See Smith (1988, 1991, 1994) or Woodford (1994).

amount of internal finance provided by domestic investors.[8] A higher domestic capital stock reduces the marginal product of capital, but it also increases the income level—and the quantity of internal finance—of domestic investors. As a result, there are two ways in which domestic borrowers can offer the necessary expected return on loans. They can either have a high capital stock and income level, a large volume of internal finance, and a low marginal product of capital, or have a low capital stock, a low volume of internal finance, and a large marginal product of capital. Hence there will typically be two steady state equilibria: one with a high, and one with a low capital stock.

The use of monetary expansions to fund government lending affects these equilibria in two ways. When the government increases the magnitude of its lending—or the implied subsidy on a given volume of lending—it must finance this by printing money. Thus the steady state rate of inflation rises. At the same time, the cost of funds to agents lending domestically—net of implied government subsidies—falls. Under weak conditions the interaction of these two effects implies that the real return that borrowers must offer lenders declines as the magnitude of government lending increases. In the high (low) activity steady state this is accomplished via a decline in the level of internal finance (the marginal product of capital), and a corresponding reduction (increase) in the steady state capital stock. This change in the capital stock also partially accounts for the results we obtain regarding dynamical equilibria.

Surprisingly, the consequences of expansionary monetary policy in an economy with credit rationing have received little attention in the literature. Azariadis and Smith (1996), Boyd and Smith (1998), and Huybens and Smith (1999) examine the consequences of changes in the rate of money creation in a closed economy context when credit is rationed. Thakor (1996) also considers how changes in monetary policy can affect the extent of credit rationing in a closed economy. Of those papers, only Azariadis and Smith (1996) allow for the possibility that monetary expansions are used to directly inject additional credit into the economy. Moreover, to our knowledge, the only paper that explicitly considers monetary policy in an open economy context with credit rationing is Huybens and Smith (1998). There monetary expansions do not fund direct lending. A comparison of the Huybens–Smith results with the results stated above will indicate that this makes a considerable difference.

The remainder of the paper proceeds as follows. Section 2 lays out the model environment, while Section 3 describes trade in credit and factor markets. As a point of reference, Section 4 analyzes the general equilibrium of a closed economy, and Section 5 then examines the steady state equilibria of a small open economy. Section 6 takes up dynamics, and Section 7 offers some concluding remarks.

[8] The point that the quantity of internal finance affects the rate of return perceived by external investors in the presence of a CSV problem was first made by Bernanke and Gertler (1989).

2. THE MODEL

In this section we describe the basic economic environment, along with the nature of trade in credit and factor markets. The framework builds a financial market friction—and, more specifically, a costly state verification problem—into a monetary version of Diamond's (1965) neoclassical growth model.

2.1. Environment

Consider a small open economy consisting of an infinite sequence of two-period lived, overlapping generations. Each generation is identical in size and composition and contains a continuum of agents with unit mass. Within each generation, agents are divided into two types: potential borrowers and lenders. A fraction $\alpha \in (0, 1)$ of the population is potential borrowers. Throughout, we let $t = 0, 1, \ldots$ index time.

At each date a single final good is produced using a constant returns to scale technology with capital and labor as inputs. We assume the final good to be mobile across borders, while factors of production are immobile. Let K_t denote the time t capital input and L_t denote the time t labor input of a representative firm. Then its final output is $F(K_t, L_t)$. F satisfies the following conditions: it is increasing in each argument, strictly concave, and $F(0, L) = F(K, 0) = 0$ holds, for all K, L. In addition, if $k \equiv \frac{K}{L}$ is the capital–labor ratio, and if $f(k) \equiv F(k, 1)$ denotes the intensive production function, then $f' > 0 > f''$ holds $\forall k$, and f satisfies the standard Inada conditions. Finally, we assume that the inherited capital stock at date t is used in production and that thereafter it depreciates completely.

With respect to endowments, all young agents are endowed with one unit of labor, which is supplied inelastically, and agents are retired when old. Individuals other than the old of period zero have no endowment of capital or final goods, while the initial old agents have an aggregate capital endowment of $K_0 > 0$.

Agents of all types are assumed to care only about old age consumption and, in addition, all agents are risk neutral. Thus all young period income is saved.

Potential borrowers and lenders are differentiated by the fact that each potential borrower has access to a stochastic linear technology for converting date t final goods into date $t + 1$ capital. Lenders have no access to this technology.

The capital investment technology has the following properties. First, the investment technology is indivisible: each potential borrower has one investment project which can only be operated at the scale q. In particular, $q > 0$ units of the final good invested in one project at t yield zq units of capital at $t + 1$, where z is an *iid* (across borrowers and periods) random variable, which is realized at $t + 1$. We let G denote the probability distribution of z and assume that G has a differentiable density function g with support $[0, \bar{z}]$. We let $\hat{z}$ denote the expected value of z.

The amount of capital produced by any investment project can be observed costlessly by the project owner. Any agent other than the project owner can observe the return on the project only by bearing a fixed cost of $\gamma > 0$ units of capital.[9]

2.2. *Factor Markets*

We assume that capital and labor are traded in competitive markets at each date. Thus if w_t denotes the time t real wage rate and ρ_t is the time t capital rental rate, the standard factor pricing relationships obtain:

$$\rho_t = f'(k_t) \tag{1}$$

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t). \tag{2}$$

Notice that $w'(k) > 0$ holds and, in addition, we will assume the following.

ASSUMPTION 1. $w''(k) < 0; \forall k \geq 0.$

Assumption 1 is satisfied if, for example, f is any CES production function with elasticity of substitution no less than one. Assumption 1 guarantees the uniqueness of a nontrivial steady state equilibrium in the model without money and without access to foreign capital markets.

2.3. *Credit Markets*

All young agents at t supply one unit of labor inelastically, earning the real wage rate w_t. For lenders this income is saved, in the form of either money, assets issued abroad, or loans to domestic borrowers. Under the assumption that the domestic country is small, domestic residents then take the world interest rate as given. We can think of all domestic credit extension as being intermediated in the manner described by Williamson (1986).

Potential borrowers also have young period income w_t, and we will assume that they must obtain external financing to operate their investment projects.

ASSUMPTION 2. $q > w(k_t)$ *for all "relevant" values of* k_t.

Let b_t denote the amount borrowed by the operator of a funded project (in real terms) at t; clearly

$$b_t = q - w(k_t). \tag{3}$$

If potential borrowers wish to obtain external funding—which is necessary to operate their projects—they do so by announcing loan contract terms. These

[9] That is, in verifying the project return, γ units of capital are used up. The assumption that capital is consumed in the verification process follows Bernanke and Gertler (1989).

announced contract terms are either accepted or rejected by intermediaries: borrowers whose terms are accepted then operate their projects. Following Williamson (1986, 1987), a loan contract consists of the following objects. First, there is a set of project return realizations A_t for which verification of the return occurs at t. Verification of project returns does not occur if $z \in B_t \equiv [0, \bar{z}] - A_t$.[10] Second, if $z \in A_t$, then the contractual repayment can meaningfully be made contingent on the project return. Thus if $z \in A_t$ we denote the promised payment (per unit borrowed) by $R_t(z)$. On the other hand, if $z \in B_t$ then the loan payment cannot meaningfully depend on the project return and the only incentive compatible loan contract offers an uncontingent payment of x_t (per unit borrowed) for all $z \in B_t$. All payments specified by any loan contract are in real terms.

Loan contracts offered by borrowers are either accepted or rejected by intermediaries who—without loss of generality—we can think of as making all domestic loans. Thus intermediaries take deposits, make loans, and conduct monitoring of project returns as required by the contracts they accept. We assume that any lender can establish an intermediary. Then, in equilibrium, intermediaries will be perfectly diversified, earn zero profits, and have a nonstochastic return on their portfolios. Thus they need not be monitored by their depositors.[11]

Intermediaries accept deposits taking their gross opportunity cost—r^d_{t+1} between t and $t+1$—as given, and they act as if they can obtain any desired quantity of deposits at that rate. It follows that, in the absence of reserve requirements, intermediaries are willing to accept loan contract offers yielding an expected return of at least r^d_{t+1}. Thus loan contract offers (that have any prospect of acceptance) must satisfy the expected return constraint

$$\int_{A_t} [R_t(z)b_t - \rho_{t+1}\gamma] g(z)\, dz + x_t b_t \int_{B_t} g(z)\, dz \geq r^d_{t+1} b_t. \tag{4}$$

In particular, expected repayments must at least cover the intermediary's cost of funds—$r^d_{t+1} b_t$—plus the real expected monitoring cost

$$\rho_{t+1}\gamma \int_{A_t} g(z)\, dz.$$

The expected monitoring cost depends on ρ_{t+1} because γ units of *capital* are expended when project returns are verified. Finally, since only project owners directly observe project returns, they must have the proper incentives to correctly

[10] We thus abstract from stochastic state verification. While this is a real restriction, Boyd and Smith (1994) show that the welfare gains from stochastic monitoring are trivial when realistic parameter values are assumed.

[11] Intermediation in this context is discussed by Williamson (1986). See Krasa and Villamil (1992) for a consideration of intermediaries that cannot perfectly diversify risk.

reveal when a monitoring state has occurred. The appropriate incentive constraint is

$$R_t(z) \leq x_t; \qquad z \in A_t. \tag{5}$$

In addition, the repayments specified by any contract must be feasible for the borrower, so that

$$R_t(z) \leq \frac{\rho_{t+1} z q}{b_t}; \qquad z \in A_t \tag{6}$$

$$x_t \leq \inf_{z \in B_t} \left[\frac{\rho_{t+1} z q}{b_t} \right]. \tag{7}$$

Equations (6) and (7) require that repayments never exceed the real value of the capital yielded by an investment project, which in state z is $zq\rho_{t+1}$ at $t+1$.

Borrowers, then, will maximize their own expected utility by choice of contract terms, subject to the constraints just described. Therefore, announced loan contracts at date t will be selected to maximize

$$\rho_{t+1} \hat{z} q - b_t \int_{A_t} R_t(z) g(z)\, dz - x_t b_t \int_{B_t} g(z)\, dz$$

subject to (4)–(7).

3. OPTIMAL LOAN CONTRACTS AND CREDIT RATIONING

In this section we characterize optimal loan contracts and state conditions under which credit might be rationed. In addition, we describe the expected payoffs of lenders and borrowers when credit rationing occurs.

3.1. Optimal Loan Contracts

The solution to the borrower's problem stated at the end of the previous section is well known: to offer a standard debt contract (modified for the presence of internal finance). In particular, the borrower either repays x_t (principal plus interest) or else defaults. In the latter case the lender monitors the project and retains the proceeds of the project net of monitoring costs. Formally,

PROPOSITION 1. *Suppose $q > b_t$. Then the optimal contractual loan terms satisfy*

$$R_t(z) = \frac{\rho_{t+1} z q}{b_t}; \qquad z \in A_t \tag{8}$$

$$A_t = \left[0, \frac{x_t b_t}{q\rho_{t+1}}\right) \tag{9}$$

$$r^d_{t+1} = \int_{A_t} \left[R_t(z) - \frac{\rho_{t+1}\gamma}{b_t}\right] g(z)\,dz + x_t \int_{B_t} g(z)\,dz. \tag{10}$$

The proof of Proposition 1 is standard[12] and we omit it here.

For future reference, we substitute (8) and (9) into (10). Then, the expected return received by a lender under the optimal contract is given by

$$\int_{A_t} \left[R_t(z) - \frac{\rho_{t+1}\gamma}{b_t}\right] g(z)\,dz + x_t \int_{B_t} g(z)\,dz$$

$$= x_t - \frac{\rho_{t+1}\gamma}{b_t} G\left(\frac{x_t b_t}{\rho_{t+1} q}\right) - \frac{\rho_{t+1} q}{b_t} \int_0^{\frac{x_t b_t}{\rho_{t+1} q}} G(z)\,dz \equiv \pi\left[x_t; \frac{b_t}{\rho_{t+1}}\right]. \tag{11}$$

The function π gives the expected return to the lender as a function of the gross loan rate, x_t, the amount of external finance required, b_t, and the future relative price of capital, ρ_{t+1}.

It will be useful in what follows to put some additional structure on the function π. In particular, we will assume the following.

ASSUMPTION 3. $g(z) + (\frac{\gamma}{q})g'(z) \geq 0$; *for all* $z \in [0, \bar{z}]$.

ASSUMPTION 4. $\pi_1[0, (\frac{b_t}{\rho_{t+1}})] > 0$.

Assumption 3 implies that $\pi_{11} < 0$. Thus, if Assumption 4 holds, the function π has the configuration depicted in Fig. 1. Evidently, depending on the value of b_t/ρ_{t+1}, there is a unique value of x_t which maximizes the expected return that can be offered to any lender. We will denote this value by $\hat{x}(b_t/\rho_{t+1})$, where the function $\hat{x}$ is defined implicitly by

$$\pi_1\left[\hat{x}\left(\frac{b_t}{\rho_{t+1}}\right); \frac{b_t}{\rho_{t+1}}\right] \equiv 1 - \left(\frac{\gamma}{q}\right) g\left[\hat{x}\left(\frac{b_t}{\rho_{t+1}}\right)\frac{b_t}{\rho_{t+1} q}\right] - G\left[\hat{x}\left(\frac{b_t}{\rho_{t+1}}\right)\frac{b_t}{q\rho_{t+1}}\right]$$
$$\equiv 0. \tag{12}$$

Equation (12) and Assumption 3 imply that

$$\hat{x}\left(\frac{b_t}{\rho_{t+1}}\right)\frac{b_t}{\rho_{t+1} q} \equiv \eta, \tag{13}$$

[12] See Gale and Hellwig (1985) or Williamson (1986, 1987).

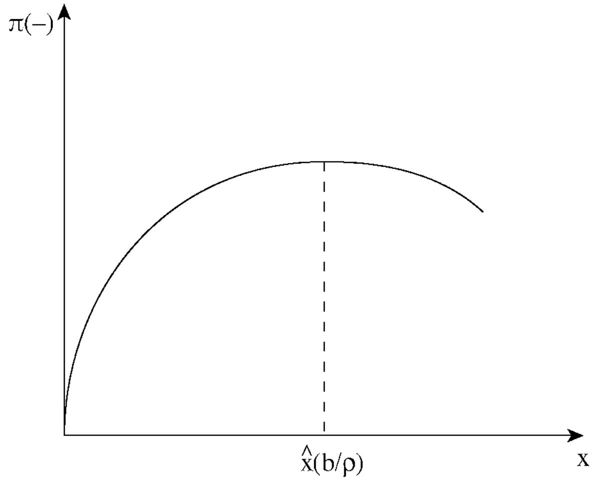

FIG. 1. The expected return function.

where $\eta > 0$ is a constant satisfying $1 - (\frac{\gamma}{q})g(\eta) - G(\eta) \equiv 0$. When all potential borrowers are offering the interest rate that maximizes a prospective lender's expected rate of return, η is the critical project return for which a borrower's project income exactly covers loan principal plus interest. In other words, project return verification occurs iff $z \in [0, \eta)$.

3.2. *Credit Rationing*

One feature of the environment under consideration—which was originally noted by Gale and Hellwig (1985) and Williamson (1986, 1987)—is that it can allow for the existence of unfulfilled demand for credit. In particular, if all borrowers desire to operate their projects at date t, the total (per capita) demand for credit is αq. The total per capita supply of saving is $w(k_t)$ at t. Domestic credit demand exceeds domestic credit supply, and hence credit must be rationed in the closed economy, if the following assumption holds for all $t \geq 0$.

ASSUMPTION 5. $\alpha q > w(k_t)$.

When credit rationing exists, however, it also must be the case that

$$x_t = \hat{x}\left(\frac{b_t}{\rho_{t+1}}\right). \tag{14}$$

When Eq. (14) holds, all potential borrowers are offering the interest rate that maximizes a prospective lender's expected rate of return. Rationed (unfunded) potential borrowers cannot then obtain credit by changing loan contract terms, since doing so simply reduces the expected return perceived by (all) lenders. Thus if Assumption 5 and Eq. (14) hold at date t, credit rationing is an equilibrium outcome for a closed economy. We henceforth focus on economies where credit rationing occurs at all dates.[13]

3.3. *Payoffs under Credit Rationing*

We now describe the expected payoffs received by lenders and (funded) borrowers at t when credit is rationed. For lenders, Eqs. (11) and (14) imply that the expected return on bank loans is given by

$$\begin{aligned}
&\pi\left[\hat{x}\left(\frac{b_t}{\rho_{t+1}}\right); \frac{b_t}{\rho_{t+1}}\right] \\
&\quad \equiv \frac{\rho_{t+1} q}{b_t}\left\{\hat{x}\left(\frac{b_t}{\rho_{t+1}}\right)\frac{b_t}{\rho_{t+1} q} - \left(\frac{\gamma}{q}\right) G\left[\hat{x}\left(\frac{b_t}{\rho_{t+1}}\right)\frac{b_t}{\rho_{t+1} q}\right] - \int_0^{\frac{\hat{x} b_t}{\rho_{t+1} q}} G(z)\,dz\right\} \\
&\quad \equiv \frac{\rho_{t+1} q}{b_t}\left[\eta - \left(\frac{\gamma}{q}\right) G(\eta) - \int_0^{\eta} G(z)\,dz\right] \equiv r_t^l,
\end{aligned} \tag{15}$$

where r_t^l denotes the gross real return on loans. Note that Eq. (15) asserts that the return to a lender between t and $t+1$ depends only on the ratio ρ_{t+1}/b_t and parameters (and is proportional to that ratio) when credit rationing obtains.

It is also straightforward to demonstrate that the expected utility of a funded borrower under credit rationing is given by

$$\rho_{t+1}\hat{z}q - r_{t+1}^l b_t - \rho_{t+1}\gamma G\left[\hat{x}\left(\frac{b_t}{\rho_{t+1}}\right)\frac{b_t}{\rho_{t+1} q}\right] \equiv \rho_{t+1} q\left[\hat{z} - \left(\frac{\gamma}{q}\right) G(\eta)\right] - r_{t+1}^l b_t.$$

Since any potential borrower always has the option of foregoing his project and depositing his income in a bank, all potential borrowers can guarantee themselves the utility level $r_{t+1}^d w_t$, where r_{t+1}^d is the rate of return on deposits between t and $t+1$. Thus (potential) borrowers prefer borrowing to lending under credit

[13] The assumption that credit rationing obtains is maintained because it results in a substantial technical simplification. However, credit rationing is clearly a widespread phenomenon in developing countries (McKinnon, 1973; Khatkate, 1982a,b; Diaz-Alejandro, 1985; Bhatt, 1988; Carter, 1988), and there is substantial evidence of significant rationing of credit even in the United States (Japelli, 1990). Therefore this does not seem to be an empirically unreasonable assumption.

rationing iff

$$\rho_{t+1} q\left[\hat{z} - \left(\frac{\gamma}{q}\right) G(\eta)\right] - r^l_{t+1} b_t \geq r^d_{t+1} w_t. \tag{16}$$

We now define

$$\phi \equiv \hat{z} - \left(\frac{\gamma}{q}\right) G(\eta)$$

$$\psi \equiv q\left[\eta - \left(\frac{\gamma}{q}\right) G(\eta) - \int_0^\eta G(z)\, dz\right]. \tag{17}$$

The variable ϕ represents the expected project yield per unit invested, under credit rationing, net of capital consumed by monitoring.[14] Similarly, ψ determines the expected rate of return received by lenders under credit rationing, since

$$r^l_{t+1} \equiv \psi \frac{\rho_{t+1}}{b_t} \equiv \psi \frac{\rho_{t+1}}{[q - w(k_t)]}. \tag{18}$$

Then (16) reduces to $q\rho_{t+1}(\phi - \psi) \geq r^d_{t+1} w_t$.

4. GENERAL EQUILIBRIUM

As a benchmark, we begin by considering a closed economy confronted with the credit market friction just described. This section also provides a number of ingredients that are useful for the analysis which follows. We start first with a nonmonetary economy and then move on to consider the behavior of a monetary economy when a reserve requirement has been imposed on agents engaged in lending activity.[15]

4.1. A Closed Economy

When credit is rationed, let μ_t denote the fraction of potential borrowers who obtain credit at t. Since each funded borrower borrows $b_t = q - w(k_t)$, total (per capita) loans are $\alpha\mu_t[q - w(k_t)]$. The total supply of savings is $(1 - \alpha)w(k_t)$ plus $\alpha(1 - \mu_t)w(k_t)$, since unfunded borrowers simply deposit their income with an intermediary and, in effect, become lenders. Thus an equality between "sources" and "uses" of funds requires that

$$\alpha\mu_t[q - w(k_t)] = (1 - \alpha)w(k_t) + \alpha(1 - \mu_t)w(k_t). \tag{19}$$

[14] Of course, we assume that $\phi > 0$.

[15] For a discussion of a closed, monetary economy, see Boyd and Smith (1998).

Equation (19) reduces to

$$\alpha q \mu_t = w(k_t). \tag{20}$$

Assumption 5 implies that $\mu_t < 1$ holds, for all $t \geq 0$.

Under our assumption that returns on investment projects are *iid* across borrowers, the fact that there is a large number of borrowers implies that there is no aggregate randomness in this economy. In particular, the time $t+1$ per capita capital stock is simply $\hat{z}\alpha q \mu_t = \hat{z} w(k_t)$, less capital expended on monitoring at $t+1$. The amount of capital consumed by monitoring is simply $\gamma \alpha \mu_t G(x_t b_t / \rho_{t+1} q) = \gamma \alpha \mu_t G(\eta) = \frac{\gamma}{q} G(\eta) w(k_t)$ under credit rationing. Thus

$$k_{t+1} = \left[\hat{z} - \left(\frac{\gamma}{q}\right) G(\eta)\right] w(k_t) = \phi w(k_t) \tag{21}$$

gives the equilibrium law of motion for k_t when credit is rationed at all dates. We will adopt the following assumption.

ASSUMPTION 6. $\phi w'(0) > 1$.

When Assumption 6 holds, Eq. (21) has the configuration depicted in Fig. 2. More specifically, given Assumption 1, k_{t+1} is an increasing, concave function of k_t. Assumption 6 implies that the locus defined by (21) lies above the 45° line for sufficiently small values of k_t, and standard arguments establish that it lies below the 45° line for large enough values of k_t. Thus Assumptions 1 and 6 imply the existence of a unique value $k^* > 0$ that satisfies $k^* = \phi w(k^*)$. Clearly, this steady

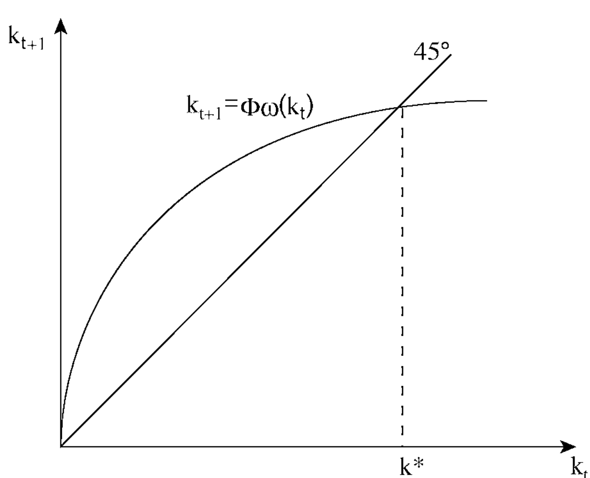

FIG. 2. The closed, nonmonetary economy.

state is asymptotically stable and dynamical equilibrium paths approaching it do so monotonically.

In order for credit rationing to obtain in the steady state, it is necessary that k^* satisfy two conditions. First,

$$\alpha q > w(k^*) \tag{22}$$

must hold so that Assumption 5 is satisfied. Second, in an economy with a single asset, the rates of return on deposits and loans must be equal. It is then easy to verify that (16) is satisfied iff

$$\phi[q - w(k^*)] \geq \psi. \tag{23}$$

If (22), (23), and $k_0 < k^*$ hold, then our economy does experience credit rationing and potential borrowers prefer borrowing over lending at all dates. It is straightforward to produce examples where (22), (23), and our other assumptions are satisfied. For instance, here is one such example.

EXAMPLE 1. Let $f(k) = Ak^{0.5}$. Then $k^* = \phi A/2$. It is easy to verify that Eqs. (22) and (23) are satisfied if $(\phi q - \psi)^{1/2} > (\phi A/2)$ and if $\psi \geq \phi q(1 - \alpha)$.[16]

4.2. A Closed Monetary Economy with a Reserve Requirement

In this section we introduce two additional factors: money and a reserve requirement that is imposed on lenders. More specifically, in this section we assume that there is a constant stock of domestic currency, issued in the amount M per capita. In addition, we assume that all agents making loans are subject to a requirement that they hold a certain quantity of this currency per dollar lent. Such reserve requirements are very common in developing countries (see, e.g., Brock (1989) and Espinosa (1994)).

Let p_t denote the (domestic) price level at time t, and let $m_t \equiv M/p_t$ be the per capita quantity of real balances. We assume that all lenders are subject to the requirement that the real balances they hold equal at least λ times the value of their loan portfolio, where the value λ is chosen by the government. Thus, in the aggregate

$$m_t \geq \lambda \alpha \mu_t [q - w(k_t)] \tag{24}$$

must hold. If $p_t/p_{t+1} < \psi(f'(k_{t+1})/[q - w(k_t)]) \equiv r_t^l$ holds, then the reserve requirement in (24) will be binding, and all lenders—who we can think of as intermediaries—will hold λ units of domestic real balances per unit lent. Hence the average return on $1 + \lambda$ units of funds deposited at t—of which one unit is lent

[16] To complete the specification of the example, suppose that $g(z) = 1/\bar{z}\ \forall z \in [0, \bar{z}]$. Then $\psi = q[\bar{z} - (\gamma/q)]^2/2\bar{z}$ and $\phi = (\psi/q) + (\gamma/q)^2/(2\bar{z})$.

and λ units are held as reserves—is given by

$$r_{t+1}^d = \frac{1}{1+\lambda}\left\{\psi \frac{f'(k_{t+1})}{[q-w(k_t)]} + \frac{p_t}{p_{t+1}}\lambda\right\}.$$

Or alternatively, defining $\theta \equiv 1/(1+\lambda)$ and $R_t^m \equiv p_t/p_{t+1}$,

$$r_{t+1}^d = \theta\psi \frac{f'(k_{t+1})}{[q-w(k_t)]} + (1-\theta)R_t^m. \tag{25}$$

Notice that $(1-\theta)$ is the fraction of assets held as required reserves. Throughout we assume that $\theta > 1/2$, so that the reserve requirement is not too large.

As before, sources of funds in this economy are simply per capita savings, $w(k_t)$, while uses of funds are investments, $\alpha q \mu_t$, plus the accumulation of real balances, m_t. Hence sources and uses of funds are equal when

$$\alpha q \mu_t + m_t = w(k_t). \tag{26}$$

Assuming that the reserve requirement is binding, Eqs. (24) and (25) imply that

$$\alpha q \mu_t = \frac{w(k_t)}{1 + \frac{\lambda}{q}[q - w(k_t)]}. \tag{27}$$

We now derive the equilibrium law of motion for the domestic capital stock. Clearly, at $t+1$, the domestic per capita capital stock is equal to $\hat{z}\alpha q \mu_t$ minus the capital expended on monitoring at $t+1$, $\gamma \alpha \mu_t G(\eta)$. It follows that the capital stock evolves according to

$$k_{t+1} = \left[\hat{z} - \frac{\gamma}{q}G(\eta)\right]\frac{w(k_t)}{1+\frac{\lambda}{q}[q-w(k_t)]} = \frac{\phi w(k_t)}{1+\frac{\lambda}{q}[q-w(k_t)]}. \tag{28}$$

If $\phi w'(0) > 1+\lambda$, then the law of motion given in (25) has the configuration depicted in Fig. 3. Under this condition there necessarily exists at least one non-trivial steady state capital stock $\bar{k}$ satisfying (28). Moreover, if we let k_l denote the largest steady state level of capital stock, it is easy to show that $k_l < k^*$ holds. It follows that Assumption 5 will be satisfied, in the steady state, if $\alpha q > w(k^*)$ obtains.

In this economy, the return on loans satisfies

$$r_{t+1}^l = \frac{\psi f'(k_{t+1})}{[q - w(k_t)]}.$$

It is easy to show that, when the reserve requirement binds, the return on real

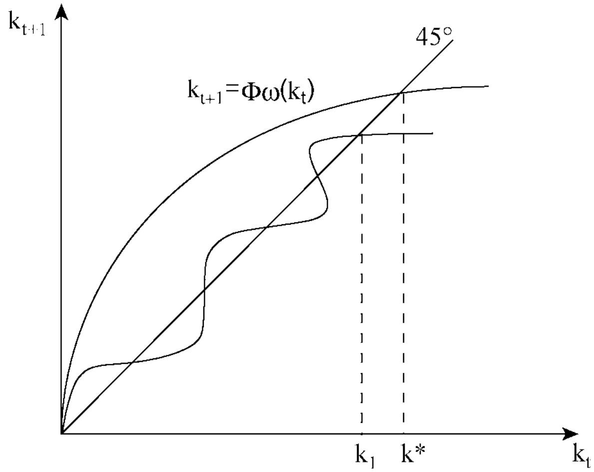

FIG. 3. The closed, monetary economy in the presence of a reserve requirement.

balances satisfies

$$R_t^m = \frac{m_{t+1}}{m_t} = \frac{k_{t+2}[q - w(k_{t+1})]}{k_{t+1}[q - w(k_t)]}.$$

The following proposition is a special case of Huybens and Smith's (1998) proposition 2:

PROPOSITION 2. (*a*) *The reserve requirement binds at t along the equilibrium path defined by* (*28*) *iff*

$$\frac{\phi w(k_{t+1})}{1 + \frac{\lambda}{q}[q - w(k_{t+1})]} \leq \psi \frac{k_{t+1} f'(k_{t+1})}{[q - w(k_{t+1})]}.$$

(*b*) *The reserve requirement binds in any steady state with credit rationing if* $\psi > f^{-1}(q)$ *holds.*

In addition, the steady state return on money in this economy is given by $R^m = 1$, while the steady state return on loans is given by $\psi f'(\bar{k})/[q - w(\bar{k})]$. Hence the reserve requirement binds in steady state iff $\psi H(\bar{k}) > 1$. As noted in part (b) of the proposition, this condition must be satisfied if $\psi H[f^{-1}(q)] = \psi / f^{-1}(q) > 1$ holds.

Finally, it remains to state conditions under which borrowers will prefer to borrow rather than lend. In a steady state equilibrium the appropriate condition is:

$$\frac{[\phi q - \psi] f'(\bar{k})}{w(\bar{k})} > \frac{\theta \psi f'(\bar{k})}{[q - w(\bar{k})]} + (1 - \theta).$$

It is straightforward to check that this condition must be satisfied if (23) holds.

5. A SMALL OPEN ECONOMY WITH GOVERNMENT CREDIT SUBSIDIES

We now turn to our primary task: to examine how government credit injections financed via an expansion of the money supply affect the equilibria of a small open economy. For obvious reasons, we focus on monetary economies. In addition we assume that the domestic government imposes a reserve requirement on all agents engaged in domestic lending. Such an assumption serves two purposes. First, for most developing countries it is realistic. Second, it enables the domestic currency to be held even though it is dominated in rate of return. Finally, the assumption that money creation is used to finance direct government lending is quite realistic for many developing countries. Moreover, it enables our model to capture an important aspect of government policy in many LDC's: the central bank is prominently engaged in domestic lending and in rediscounting the loans of private banks.

Often, the rationale for such policies is that financial market frictions lead credit to be rationed domestically. One such friction is the CSV problem which arises endogenously. We now examine how this friction interacts with the operation of international capital markets and with the lending programs conducted by the government.

5.1. *International Capital Markets*

We assume that domestic residents can borrow and lend in world capital markets at the world rate of interest R.[17] Since the domestic country is assumed to be small, the activity of its residents or its government does not affect this rate.

As before, we assume that all agents lending domestically are subject to a reserve requirement that, for every dollar lent, λ dollars of domestic currency must be held. Hence, if $m_t \equiv M_t/p_t$ denotes the per capita supply of domestic real balances, in the aggregate m_t must continue to satisfy Eq. (24). If $R > 1$ holds, then clearly the domestic reserve requirement will be binding. In addition, we assume that the domestic country operates under a regime of flexible exchange rates, so that it is free to finance its expenditures via money creation.

5.2. *Government Credit Policy*

As we already noted, monetary policy is conducted by rediscounting the loans of private sector banks. More specifically, for every dollar of private sector *lending*, the central bank extends $\tau \in [0, 1)$ dollars of credit to private banks. Given the presence of binding reserve requirements in our economy, this is equivalent to the assumption that the government lends $\tau\theta$ dollars to each bank per dollar of assets held.

On each unit that the government lends, it charges the gross real rate of interest r. We assume that $r \leq 1$ holds and therefore, $R > 1 \geq r$ holds as well. We do *not*

[17] R is a gross real rate. Also, foreigners who lend in the domestic economy are subject to domestic reserve requirements.

impose that $r \geq R_t^m$ must be satisfied: if this condition fails the central bank charges negative nominal rates of interest at t. This can be interpreted as the government extending credit in combination with a subsidy. In the extreme case $r = 0$, the government is simply offering a lump-sum transfer to each bank. Note that both r and τ are policy parameters that are selected by the government.

In order to finance its lending activity, the central bank issues money. At date t, let μ_t denote the fraction of domestic borrowers who receive loans. Then the private sector extends loans with a real value of $\alpha\mu_t[q - w(k_t)]$ at t. The central bank finances a fraction τ of this lending; hence its loans at t equal $\tau\alpha\mu_t[q - w(k_t)]$. Similarly when the government extends loans of $\tau\alpha\mu_{t-1}[q - w(k_{t-1})]$ at $t - 1$, its interest income at t equals $r\tau\alpha\mu_{t-1}[q - w(k_{t-1})]$. The difference between time t loans and time t interest income must be financed with seigniorage income. Hence, the time t government budget constraint is given by

$$\frac{M_t - M_{t-1}}{p_t} = \tau\alpha\mu_t[q - w(k_t)] - r\tau\alpha\mu_{t-1}[q - w(k_{t-1})]. \tag{29}$$

Clearly, when $\tau = 0$, the central bank is inactive, and the money stock is constant.

5.3. General Equilibrium

Since $R > 1$ holds, the reserve requirement necessarily binds. Hence (24) holds with equality. That condition, along with the government budget constraint implies that

$$R_t^m = r\tau\left(\frac{\theta}{1-\theta}\right) + \left[1 - \tau\left(\frac{\theta}{1-\theta}\right)\right]\frac{k_{t+2}[q - w(k_{t+1})]}{k_{t+1}[q - w(k_t)]}. \tag{30}$$

In addition, domestic lenders hold portfolios with a weight θ attached to loans, and a weight $(1 - \theta)$ attached to reserves. Hence the return on a bank's assets is given by

$$\theta\psi\frac{f'(k_{t+1})}{[q - w(k_t)]} + (1-\theta)R_t^m.$$

Each bank raises funds by borrowing a fraction $(1 - \tau\theta)$ of its funds in private capital markets, and it borrows $\tau\theta$ from the government. Hence any bank's cost of funds equals $(1 - \tau\theta)R + \tau\theta r$. If there is free entry into domestic lending each bank must earn zero profits; hence in equilibrium,

$$\theta\psi\left[\frac{f'(k_{t+1})}{q - w(k_t)}\right] + (1-\theta)R_t^m = (1 - \tau\theta)R + \tau\theta r; \qquad t \geq 1, \tag{31}$$

must obtain.

Equations (30) and (31) govern the evolution of the domestic capital stock, $\{k_t\}$. Having obtained this sequence, Eq. (30) gives the equilibrium inverse inflation rate, R_t^m. Finally, sources and uses of funds must be equal. Sources of funds are domestic per capita savings $w(k_t)$. Uses of funds are domestic investments, $\alpha q \mu_t$, plus real balances m_t, plus net claims acquired abroad, s_t. Hence

$$s_t = w(k_t) - \alpha q \mu_t - m_t = w(k_t) - (k_{t+1}/\phi) - m_t. \tag{32}$$

Equation (32) allows one to deduce the net flows of capital into and out of the domestic country.

5.4. *Steady State Equilibria*

We begin our analysis of equilibria by examining steady states. Dynamical equilibria are studied in Section 6.

In steady state, the government budget constraint reduces to

$$m(1 - R^m) = (1 - r)\tau\alpha\mu[q - w(k)]. \tag{33}$$

In addition when the reserve requirement binds, we have

$$m = \lambda\alpha\mu[q - w(k)] = \left(\frac{1-\theta}{\theta}\right)\alpha\mu[q - w(k)]. \tag{34}$$

Equations (33) and (34) imply that the steady state return on reserves is given by

$$R^m = 1 - (1 - r)\tau\left(\frac{\theta}{1-\theta}\right). \tag{35}$$

If we now define the function $H(k)$ by

$$H(k) = \frac{f'(k)}{[q - w(k)]},$$

then (31) and (35) imply that steady state capital stock satisfies the condition

$$\theta\psi H(k) + (1-\theta)\left[1 - (1-r)\tau\left(\frac{\theta}{1-\theta}\right)\right] = (1 - \tau\theta)R + \tau\theta r \tag{36}$$

or equivalently,

$$\theta\psi H(k) = R - (1-\theta) - \tau\theta(R - 1). \tag{37}$$

One interesting result follows immediately from Eq. (37): the steady state capital stock or capital stocks do *not* depend in any way on the rate of interest that the central bank charges on the loans it makes.[18] Intuitively, while charging a below market rate of interest on these loans does represent a subsidy to banks, this subsidy is financed by an inflation tax that banks pay on their reserve holdings. In equilibrium these effects cancel each other out, and the choice of the rate of interest r is irrelevant to the level of real activity.

In order to provide a further characterization of steady states, evidently it will be useful to know more about the function $H(k)$. Some of its properties are stated in the following lemma.

LEMMA 1. *The function H satisfies*

(a) $\lim_{k\to 0} H(k) = \infty$
(b) $\lim_{k\to \hat{k}} H(k) = \infty$, *where* $\hat{k} \equiv w^{-1}(q)$
(c) $H'(k) \le 0;\ k \le f^{-1}(q)$, *and*
(d) $H'(k) \ge 0;\ k \ge f^{-1}(q)$.

Proof. Part (a) is, of course, immediate from the Inada conditions and $w(0) = 0$. Part (b) is also obvious. For (c) and (d), straightforward differentiation yields

$$H'(k) = -f''(k)\frac{[f(k) - q]}{[q - w(k)]^2},$$

establishing the result. ■

Lemma 1 implies that the function H has the configuration depicted in Fig. 4. The following proposition is now immediate.

PROPOSITION 3. (a) *Suppose that*

$$\left[\frac{R - (1-\theta) - \tau\theta(R-1)}{\theta\psi}\right] > H[f^{-1}(q)]. \tag{38}$$

Then there are exactly two values of k, denoted by k_1 and k_2 in Fig. 4, that satisfy (37).

(b) *Suppose that*

$$\left[\frac{R - (1-\theta) - \tau\theta(R-1)}{\theta\psi}\right] < H[f^{-1}(q)]. \tag{39}$$

Then there is no monetary steady state with credit rationing in the presence of foreign asset flows and a reserve requirement.

[18] As we will show subsequently, this is also true of the entire dynamical equilibrium path $\{k_t\}$. We also note that this result depends on our assumptions about how government lending is financed and also on the fact that here all base money is held as bank reserves. We discuss this point in greater detail below.

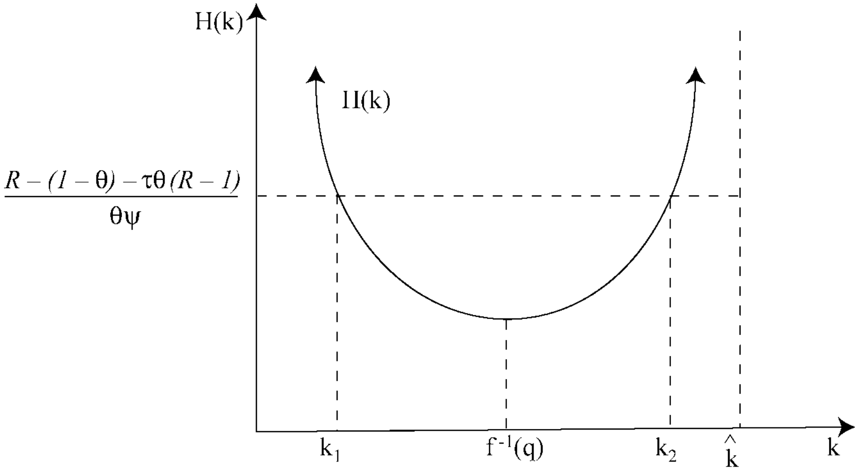

FIG. 4. The open monetary economy in the presence of a reserve requirement.

When Eq. (38) is satisfied there are two candidate steady state equilibria. Their associated capital stocks are denoted by k_1 and k_2 in Fig. 4. In addition, in order to verify that credit is rationed in each steady state, it is necessary to check that the solutions to (37) imply that $\mu < 1$ holds. It is easy to show that $\mu < 1$ in each steady state iff $k_2/\phi\alpha q < 1$. It is also necessary to verify that $R^m < R$ holds, so that the reserve requirement binds. This will be the case if $R > 1$, although clearly this is sufficient rather than necessary for the reserve requirement to bind. Finally, we must check that borrowers prefer borrowing to lending in each candidate steady state. It is straightforward to verify that this requirement of equilibrium will be satisfied at each candidate steady state if $(\phi q - \psi)f'(k_2)/w(k_2) > R$.

Parenthetically, one may ask how credit rationing can arise in a small open economy, which presumably can absorb large quantities of funds without affecting world capital markets? The answer derives from the presence of the CSV problem. At date t, the domestic economy has an inherited capital stock, k_t. Given k_t— and $w(k_t)$—and given the perfect foresight domestic price level sequence $\{p_t\}$, domestic borrowers must offer a return on loans no less than $R - (1-\theta)R^m$ in order to compete for funds. The highest expected return that domestic agents can offer at $t+1$ is $\psi f'(k_{t+1})/[q - w(k_t)]$. Thus, given k_t and R_t^m, there is a largest value of k_{t+1} that permits domestic borrowers to compete in international financial markets. Credit rationing arises, then, because any further domestic credit extension would be inconsistent with domestic borrowers offering a competitive rate of return on loans.[19]

[19] For a small open economy there may also be equilibria in which credit is not rationed (see Huybens and Smith (1998)). It is easy to describe conditions under which equilibria without credit rationing have the feature that all domestic investment is self-financed. When this is the case, any lending

We close this section with an example of an economy that satisfies all of our assumptions. The example is identical to Example 1, except for the presence of the central bank lending scheme.

EXAMPLE 2. Again let $f(k) = Ak^{0.5}$. In addition, let $\theta = .9$ and $\tau = 1/9$. Then it is straightforward to verify that the steady state equilibrium capital stocks satisfy[20]

$$\sqrt{k} = \frac{q}{A} \pm \sqrt{\left(\frac{q}{A}\right)^2 - \left(\frac{\psi}{R}\right)}. \tag{40}$$

It is also straightforward to verify that borrowers prefer to borrow rather than lend at each steady state iff

$$\left(\frac{\phi q - \psi}{R}\right)^{1/2} > \left(\frac{q}{A}\right) + \left[\left(\frac{q}{A}\right)^2 - \left(\frac{\psi}{R}\right)\right]^{1/2}. \tag{41}$$

Moreover, if (41) and $\psi \geq \phi q(1 - \alpha R)$ both hold, then there is rationing of credit in both states (that is, $k_2 < \phi\alpha q$).

5.5. *Comparative Statics*

We now examine the comparative static consequences of changes in world financial market conditions and of changes in central bank policy. We focus particularly on the capital stock and, by implication, long-run real activity in our analysis. The effect of an increase in the world interest rate, R, depends on which steady state obtains, as illustrated in Fig. 5. In the low-capital-stock steady state, an increase in the world interest rate from R to R' reduces the steady state capital stock, while the same increase raises the capital stock in the high-capital-stock steady state. Intuitively, when the world interest rate rises, the return on domestic investments must also rise in order for domestic borrowers to compete in international capital markets. In the low (high) capital-stock steady state, an increase in the marginal product of capital (an increase in the level of internal finance) is required to accomplish this, so that the capital stock must fall (rise).

An increase in τ represents a policy of more active rediscounting by the central bank and, by implication, a higher level of credit subsidization. The effect of an increase once again depends on which steady state prevails, as shown in Fig. 6. Since $R > 1$ holds, an increase in τ has the effect of reducing (increasing) the

activity—through the central bank or otherwise—will obviously be irrelevant. For this reason, we confine our attention to equilibria in which credit is rationed.

[20] It is easy to show that when $\theta = .9$ (a reserve requirement of 10% is in place), setting $\tau = 1/9$ exactly "undoes" the effect of the reserve requirement on the steady state capital stocks. Or, in other words, the steady state capital stocks coincide with those that would obtain in the absence of a reserve requirement.

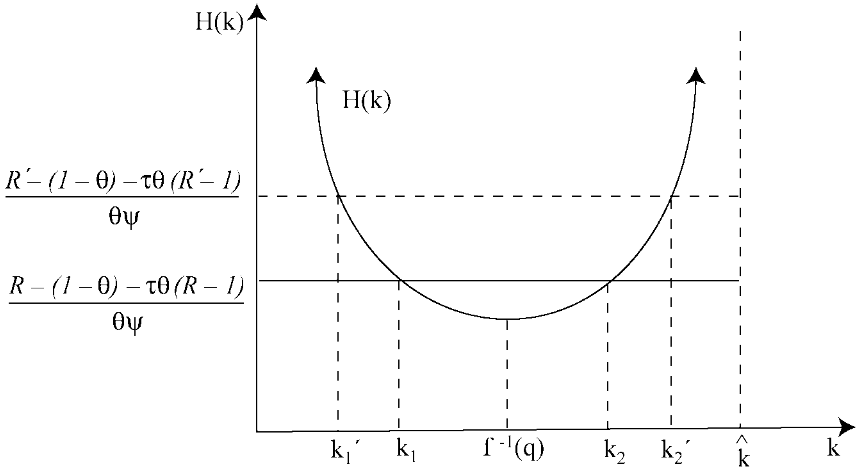

FIG. 5. The effect of an increase in the world real interest rate.

capital stock in the high (low)-capital-stock steady state. In effect, the increase in the size of the government subsidy program gives banks access to cheaper sources of funds. As a result of competition, the zero profit condition implies that the return on bank loans must fall; in the high (low)-capital-stock steady state this requires a reduction in the level of internal finance (an increase in the marginal product of capital). Thus the capital stock must fall (rise).

It has often been observed that subsidizing lending has counterproductive consequences in developing countries where credit rationing is prevalent (see, for

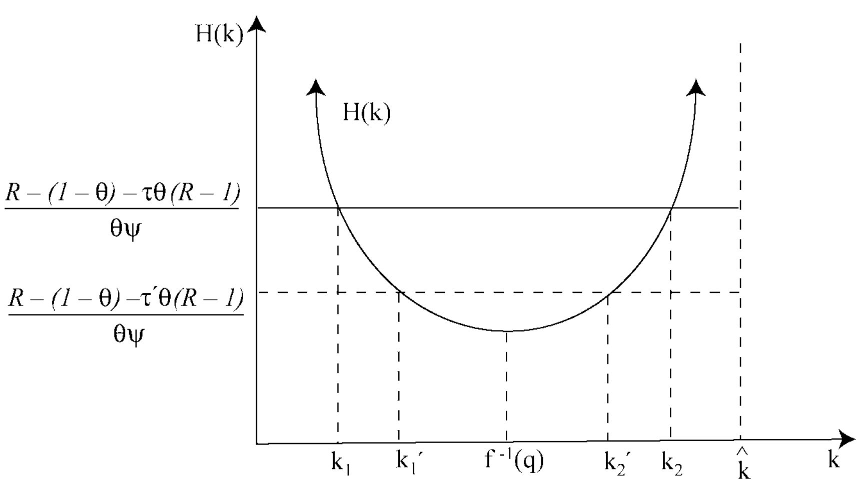

FIG. 6. The effect of an increase in the fraction of lending financed by the central bank.

example, the references in footnote 6). Our results provide some circumstances under which one would expect this to be the case.

5.6. Discussion

The results just described are based on Eq. (37). The exact form of that equilibrium condition depends on two features of our analysis: the assumed method by which central bank lending is financed and the fact that all base money is held by banks as required reserves. It is useful to explore the extent to which our results are dependent on these features of the model.

In order to do so, it is useful to think about a slightly different type of central bank policy than the one described above. In particular, suppose that the central bank exogenously selects two parameters: a gross rate of money creation, σ, and a fraction η of seigniorage revenue that is lent to banks at a subsidized rate $r < R$. This specification of policy allows us to consider changes in government lending and changes in the steady state rate of inflation separately.

In addition, to explore what might happen if some of the monetary base is held in a form other than bank reserves, we assume that some exogenously given fraction of lenders has no access to intermediaries.[21] These agents then save in the form of fiat money. For simplicity we assume that the ratio of currency in the hands of the public to deposits, c, is constant.

Under these assumptions, it is easy to show that the analog to (37) is

$$\theta\psi H(k) + (1-\theta)\left(\frac{1}{\sigma}\right) = R - \eta(R-r)\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{m_t}{d_t}\right),$$

where d_t denotes the real value of bank deposits.[22]

It is also easy to see that, if $\sigma > 1$ (there is positive money growth), $\partial k/\partial\eta <$ ($>$)0 holds at the high (low)-capital-stock steady state. Hence our result that an expansion in subsidized lending lowers the capital stock in the high activity steady state is robust; it does not depend on the features of the model identified above.

On the other hand, the effects of an increase in the money growth rate, σ, are substantially more complicated in this modified version of our model. In particular, it is possible to show that $\partial k/\partial\sigma < (>)0$ holds at the high activity steady state iff

$$(1-\theta) < (>)(R-r)\eta\left(\frac{m_t}{d_t}\right).$$

Thus $\partial k/\partial\sigma < 0$ (higher rates of inflation inhibit long-run capital formation) at the high activity steady state if (a) the reserve requirement $(1-\theta)$ is relatively low, (b) central bank lending enjoys relatively high subsidies ($[R-r]$ is relatively

[21] This is a very realistic assumption for a typical developing economy.

[22] The assumption of a binding reserve requirement and a constant currency–deposit ratio imply that m_t/d_t is a constant.

large), (c) the discount window is relatively active (η is relatively large), and (d) the currency–deposit ratio is relatively large. Thus the effects of higher rate of inflation depend significantly on several parameter values when we modify our economy in the manner described. Nonetheless, our key conclusion remains intact: it can easily happen that using expansionary monetary policy to increase the injection of credit into an economy (that is, increasing η and σ simultaneously) can result in a reduction in long-run real economic activity, unless the economy is in a development trap.

6. DYNAMIC EQUILIBRIA

When the reserve requirement binds, Eqs. (30) and (31) can be used to derive the following equilibrium law of motion for the capital stock,

$$k_{t+2} = B(\tau)^{-1} k_{t+1} \left[\frac{q - w(k_t)}{q - w(k_{t+1})} \right] \left[A(\tau) - \frac{\theta \psi f'(k_{t+1})}{q - w(k_t)} \right], \tag{42}$$

where $A(\tau) \equiv (1 - \theta\tau)R > 0$ and $B(\tau) \equiv 1 - (1 + \tau)\theta$. Clearly the evolution of the capital stock is governed by a second order difference equation. We now proceed as follows.

Defining

$$k_{t+1} = y_t, \tag{43}$$

we may rewrite Eq. (42) as

$$y_{t+1} = B(\tau)^{-1} y_t \left[\frac{q - w(k_t)}{q - w(y_t)} \right] \left[A(\tau) - \frac{\theta \psi f'(y_t)}{q - w(k_t)} \right]. \tag{44}$$

We henceforth work with the dynamical system consisting of Eqs. (43) and (44).

In order to analyze local dynamics, we linearize (43) and (44) in a neighborhood of any steady state, $(k, \; y)$. We then have

$$(k_{t+1} - k, y_{t+1} - y)' = J(k_t - k, y_t - y)',$$

where J is the Jacobian matrix

$$J = \begin{bmatrix} \frac{\partial k_{t+1}}{\partial k_t} & \frac{\partial k_{t+1}}{\partial y_t} \\ \frac{\partial y_{t+1}}{\partial k_t} & \frac{\partial y_{t+1}}{\partial y_t} \end{bmatrix}$$

with all partial derivatives evaluated at the appropriate steady state. It is straight forward to show that

$$\frac{\partial k_{t+1}}{\partial k_t} = 0, \tag{45}$$

$$\frac{\partial k_{t+1}}{\partial y_t} = 1, \tag{46}$$

$$\frac{\partial y_{t+1}}{\partial k_t} = -A(\tau)B(\tau)^{-1}\left[\frac{kw'(k)}{q - w(k)}\right], \tag{47}$$

$$\frac{\partial y_{t+1}}{\partial y_t} = 1 + \left[\frac{kw'(k)}{q - w(k)}\right]\left[1 + \frac{\theta\psi}{kB(\tau)}\right]. \tag{48}$$

Let $T(k)$ and $D(k)$ be the trace and determinant of J, respectively. Then we have

$$T(k) = 1 + \left[\frac{kw'(k)}{q - w(k)}\right]\left[1 + \frac{\theta\psi}{kB(\tau)}\right], \tag{49}$$

$$D(k) = A(\tau)B(\tau)^{-1}\left[\frac{kw'(k)}{q - w(k)}\right]. \tag{50}$$

Notice that (49) can be rewritten as

$$T(k) = 1 + A(\tau)^{-1}B(\tau)D(k)\left[1 + \frac{\theta\psi}{kB(\tau)}\right]. \tag{51}$$

Now, we would like to analyze equilibrium dynamics in a neighborhood of each steady state. The nature of these dynamics depend upon the signs of $T(k)$ and $D(k)$, which in turn depend upon the sign of $B(\tau) \equiv 1 - (1 + \tau)\theta$. Clearly, $B(\tau) > (<) 0$ holds if $\tau < (>) \frac{1-\theta}{\theta}$. Thus, given the reserve requirement $(1 - \theta)$, $B(\tau)$ is positive (negative) when government credit provision is relatively small (large). We next consider each case separately.

6.1. *Case 1: $B(\tau) > 0$*

When $B(\tau) > 0$ holds, we have $D(k) > 0$ and $T(k) > 1$. Hence the eigenvalues of J are either both positive real numbers, or they are complex conjugates. In addition, it is possible to derive very definite conclusions about the local stability properties of each steady state. We begin with the following result.

PROPOSITION 4. *When $B(\tau) > 0$ holds, $T(k) \gtreqless 1 + D(k)$ iff $q \gtreqless f(k)$.*

Proof. Using the definitions of $A(\tau)$ and $B(\tau)$, we rewrite (36) as

$$\theta\psi H(k) = A(\tau) - B(\tau). \tag{52}$$

Equations (51) and (52) then imply that

$$1 + D(k) - T(k) = \frac{\theta \psi D(k)}{A(\tau)k}[kH(k) - 1] \tag{53}$$

holds. Since $kH(k) = kf'/(kf' + [q - f(k)])$, it follows that $kH(k) \gtreqless 1$ iff $q \gtreqless f(k)$. ■

Proposition 4 has an immediate corollary: the low-capital-stock steady state is necessarily a saddle, while the high-capital-stock steady state is either a source or a sink. Specifically, it is a sink (source) when $D(k) < (>)1$.[23] These observations have the following implications. First, since there is a single given initial condition (the initial capital stock), there is a unique dynamical equilibrium path approaching the low-capital-stock steady state. Second, local dynamics near the low-capital-stock steady state are necessarily monotone. Third, since there is a single initial condition, when the high-capital-stock steady state is a sink there is an indeterminacy of equilibrium: there are many dynamical equilibria that approach the high activity steady state. We now turn our attention to when the high-capital-stock steady state is and is not a sink and to the nature of dynamical equilibrium paths in its vicinity.

In order to obtain sharp answers to these questions, we henceforth assume that the production function takes the Cobb–Douglas form $f(k) = Ak^{\delta}$, with $\delta \in (0, 1)$. Under this assumption, $D(k_2)$ has a particularly simple form:

$$D(k_2) = (1 - \delta)A(\tau)B(\tau)^{-1}k_2H(k_2). \tag{54}$$

Then, in particular, $D(k_2) < (>)1$ holds iff $(1 - \delta)A(\tau)B(\tau)^{-1}k_2H(k_2) < (>)1$ obtains. We can now state an immediate implication of this observation.

PROPOSITION 5. *The high-capital-stock steady state is a source if*

$$(1 - \delta)A(\tau)B(\tau)^{-1} > 1.$$

Proof. Since $k_2 \geq f^{-1}(q)$, $k_2H(k_2) \geq H[f^{-1}(q)]f^{-1}(q) \equiv 1$ must hold. Thus $D(k_2) \geq (1 - \delta)A(\tau)B(\tau)^{-1}$, which establishes the result. ■

Since as $\tau \uparrow (\frac{1-\theta}{\theta})$, $A(\tau)B(\tau)^{-1} \to \infty$, it is apparent that the high-capital-stock steady state will be a source if there is not too much lending by the central bank. Once again the inflationary consequences of using money creation to fund lending are largely the culprit: the high rates of inflation caused by printing money will imply that the high-capital-stock steady state cannot be approached.

[23] See Azariadis (1993, Chap. 6.4).

We now recall that there are two steady state equilibria with credit rationing iff (38) holds. Suppose that[24]

$$\text{(i)}\quad R > (1-\theta) + \theta\left[\frac{\psi}{f^{-1}(q)}\right]$$

holds, and define the value τ^* by

$$\tau^* \equiv \frac{R-(1-\theta)-\theta\frac{\psi}{f^{-1}(q)}}{\theta(R-1)}.$$

If we also assume that

$$\text{(ii)}\quad (1-\theta)(R-1) < \left[\frac{\psi}{f^{-1}(q)} - 1\right]\theta$$

then $\tau^* \in (0, 1)$. Condition (ii) will hold if $\psi H[f^{-1}(q)] > 1$[25] and if either the reserve requirement or the world rate of interest is not too large.

When $\tau^* < 1$ is satisfied, it is easy to verify that (38) [39] holds if $\tau < (>)\tau^*$. Stated differently, there are two nontrivial steady state equilibria with credit rationing iff $\tau < \tau^*$ is satisfied. We henceforth assume that this condition obtains. It is also useful to know when $\tau^* < (>)\ \frac{1-\theta}{\theta}$ holds, since if $\tau^* < \frac{1-\theta}{\theta}$, $B(\tau) > 0$ is satisfied for all $\tau \leq \tau^*$. It is straightforward to verify that $\tau^* < (>)\ \frac{1-\theta}{\theta}$ obtains iff

$$\text{(iii)}\quad \psi > (<) Rf^{-1}(q).$$

Condition (iii) implies that $\tau^* < (>)\frac{1-\theta}{\theta}$ holds if $\psi H[f^{-1}(q)] > (<)R$ or, in other words, if the minimum steady state real rate of return on loans does (does not) exceed the world real interest rate. Of course if $\tau^* > \frac{1-\theta}{\theta}$, then the government can set τ in such a way that two steady state equilibria with credit rationing exist, and that $B(\tau) < 0$.

Armed with conditions on τ such that (38) holds, we are now prepared to state results concerning how the stability properties of the high-capital-stock steady state depend on the magnitude of government credit extension. We begin with the following proposition.

PROPOSITION 6. *Suppose $\tau^* < \frac{1-\theta}{\theta}$ holds. Then for τ sufficiently close to, but below τ^*, $D(k_2) < (>)1$ holds if*

$$\text{(iv)}\quad \left[\frac{\psi}{f^{-1}(q)}\right][1-(1-\delta)R] > (<)\delta R.$$

[24] Condition (i) asserts that there must be two steady state equilibria with credit rationing when $R^m = 1$ holds.

[25] This condition implies that the net real rate of return on loans is always positive, and hence the reserve requirement always binds.

The proofs of Propositions 6, 7, and 8 can be found in the Appendix. Proposition 6 describes conditions under which large volumes of central bank lending lead the high-capital steady state to be a sink (source). When it is a sink (source), there is an indeterminacy of equilibrium (it is not possible to approach the high-capital-stock steady state). The next proposition states similar results when the volume of central bank lending is small.

PROPOSITION 7. *(a) When $\tau = 0 < \tau^*$ holds, the high-capital-stock steady state is a sink if*

$$\text{(v)} \quad \left[\frac{(1-\delta)R}{(1-\theta)}\right]\left[\frac{R-(1-\theta)}{\theta\psi}\right]\phi\alpha q \leq 1$$

and if credit is rationed ($\mu < 1$).

(b) When $\tau = 0$, the high-capital-stock steady state is a source if $(1-\delta)R > (1-\theta)$.

When (v) holds, the high-capital-stock steady state is a sink whenever the government credit program is sufficiently small.

Finally, we state conditions under which $B(\tau) > 0$ holds and under which dynamical equilibrium paths approaching the high-capital-stock steady state display damped oscillation. Our results also imply that, depending on the choice of τ, this endogenously arising volatility may dampen only arbitrarily slowly. Thus long-lived, endogenously generated volatility can easily be observed for some settings of government policy.

PROPOSITION 8. *Suppose that condition (v) and $\tau^* > \frac{1-\theta}{\theta}$ hold. (a) Then there exists a value $\tau_c \in (0, \frac{1-\theta}{\theta})$ such that, when $\tau = \tau_c$, $D(k_2) = 1$ holds. (b) For τ in a neighborhood of τ_c, $T(k_2)^2 < 4D(k_2)$ holds. Consequently, the eigenvalues of J are complex conjugates.*

Proposition 8 indicates that when the volume of the central bank lending is close enough to—but below τ_c—the high-capital-steady state is a sink with all that implies. Moreover, dynamical equilibrium paths approaching that steady state can display fluctuations that dampen only arbitrarily slowly. Thus sufficiently large credit injections by the central bank will necessarily be a source of highly persistent and endogenously arising economic volatility.

Obviously, all of the preceding results have been derived when $\tau < (\frac{1-\theta}{\theta})$ $[B(\tau) > 0]$. Assuming $\tau^* > \frac{1-\theta}{\theta}$ holds, we now consider the properties of dynamical equilibria for values of τ such that $\tau > \frac{1-\theta}{\theta}$.

6.2. *Case 2: $B(\tau) < 0$*

When the volume of the central bank rediscounting is sufficiently large, $B(\tau) < 0$ is satisfied. Moreover, whenever $B(\tau) < 0$ holds, we have $D(k) < 0$. It follows that J has two real eigenvalues of opposite signs. The local stability properties of the two steady states are dramatically altered relative to the $B(\tau) > 0$ case. For notational purposes, let λ_1 and λ_2 denote the eigenvalues of J, and let $\lambda_1 > 0 > \lambda_2$. Then we have the following results.

LEMMA 2. *Let $B(\tau) < 0$ hold. Then we have $1 + D(k_2) < T(k_2)$ so that $\lambda_1 > 1$.*

Proof. For $B(\tau) < 0$, Eq. (53) and $k_2 H(k_2) \geq 1$ imply that $1 + D(k_2) - T(k_2) < 0$. ■

The lemma implies that when $\tau > \frac{1-\theta}{\theta}$, the high-capital-stock steady state cannot be a sink. It must be either a source or a saddle. If it is a saddle, then dynamical equilibria approaching the high-capital-stock steady state are determinate. This presents a sharp contrast with the case where $B(\tau) > 0$. We can also establish that the low-capital-stock steady state cannot be a source. It must be either a sink or a saddle.

LEMMA 3. *Let $B(\tau) < 0$ hold. Then we have $1 + D(k_1) > T(k_1)$ so that $\lambda_1 < 1$.*

Proof. For $B(\tau) < 0$, $1 + D(k_1) - T(k_1) > 0$ follows directly from (53) and $k_1 H(k_1) < f^{-1}(q) H[f^{-1}(q)] = 1$. ■

It remains to study the magnitude of the negative eigenvalues at each steady state. The following lemmas provide some useful information under the Cobb–Douglas production specification.

LEMMA 4. *If $1 + A(\tau)B(\tau)^{-1} \geq 0$, then $\lambda_2 > -1$.*

Proof. From (51), we have

$$1 + D(k) + T(k) = (1 + A(\tau)^{-1} B(\tau))(1 - \delta)[1 + kH(k)] + 2\delta. \tag{55}$$

If $1 + A(\tau)B(\tau)^{-1} \geq 0$, then $1 + D(k) + T(k) > 0$ is satisfied, and the result follows [Azariadis (1993, Chap. 6.4)]. ■

Lemma 4 states a condition under which the high-capital-stock steady state is a saddle ($-1 < \lambda_2 < 0 < 1 < \lambda_1$). Thus the local stability properties of the steady states can be reversed when $B(\tau) < 0$. In particular, there are indeterminacies in a neighborhood of the low-capital-stock steady state when $B(\tau) < 0$ and $1 + A(\tau)B(\tau)^{-1} \geq 0$. Under the same conditions, there is a unique equilibrium path approaching the high-capital-stock steady state. At the same time, paths approaching that steady state must display endogenously arising fluctuations as they do so. As we show below, for appropriate choices of τ, those fluctuations will dampen only arbitrarily slowly.

Lemma 4 implies that government credit policies can potentially be used to render equilibria in a neighborhood of the high-capital-stock steady state determinate. This is clearly an argument in favor of such policies. However, determinacy comes at the price of allowing endogenously arising volatility to emerge. Moreover, determinacy requires that the central bank be rediscounting a relatively large volume of loans. This, in turn, will imply that the capital stock and the level of real activity at the high-capital steady state are relatively low.

When exactly can government policy be used to induce determinacy of equilibrium in a neighborhood of the high-capital-stock steady state? The definitions of $A(\tau)$ and $B(\tau)$ imply that $1 + A(\tau)B(\tau)^{-1} \geq 0$ holds iff

$$(1+\tau)\theta \geq 1 + (1-\theta\tau)R. \tag{56}$$

Equation (56) requires that both θ and τ be relatively large. In other words, the high-capital-stock steady state will be a saddle if the reserve requirement $(1-\theta)$ is sufficiently low, and if the volume of central bank lending (τ) is relatively large.

Not surprisingly, it is also possible to show the converse: when θ and τ are relatively small, consistent with $B(\tau) < 0$, the high-capital-stock steady state is a source. We now state the relevant result.

LEMMA 5. *If* $-[1 + A(\tau)B(\tau)^{-1}] > \delta/(1-\delta)$, *then* $\lambda_2 < -1$ *at the high-capital-stock steady state.*

Proof. Equations (55) and $k_2 H(k_2) \geq 1$ imply that $1 + D(k_2) + T(k_2) < 0$ necessarily holds if $-[1 + A(\tau)B(\tau)^{-1}] \geq \delta/(1-\delta)$. The result then follows [Azariadis (1993, Chap. 6.4)]. ■

The fact that $\lambda_2 > (<) -1$ holds at the high-capital-stock steady state when τ is large (small) suggests the existence of a critical value, denoted $\hat{\tau}$, such that, when $\tau = \hat{\tau}$, $\lambda_2 = -1$ at the high-capital-stock steady state. The next lemma confirms this intuition. It is then immediate that, for some choices of τ, the high-capital-stock steady state is a saddle and that paths that approach it display oscillation that dampens arbitrarily slowly.

LEMMA 6. *Suppose that* $\theta \geq \frac{1+R}{2+R}$ *holds. Then there exists a value* $\tilde{\tau} \in (\frac{1-\theta}{\theta}, 1)$ *such that* $1 + A(\tau)B(\tau)^{-1} = 0$. *In addition, there exists a value* $\hat{\tau} \in (\frac{1-\theta}{\theta}, \tilde{\tau})$ *such that, when* $\tau = \hat{\tau}$, $\lambda_2 = -1$ *at the high-capital-stock steady state.*

Proof. Since

$$A(\tau)B(\tau)^{-1} \equiv \frac{(1-\tau\theta)R}{1-\theta(1+\tau)},$$

it follows that

$$\frac{d[A(\tau)B(\tau)^{-1}]}{d\tau} = \frac{R\theta^2}{B(\tau)^2} > 0.$$

Moreover, it is straightforward to calculate that $1 + A(\tau)B(\tau)^{-1} = 0$ holds iff

$$\tau = \frac{R + (1-\theta)}{\theta(1+R)} \equiv \tilde{\tau}$$

and that under our hypotheses, $\tilde{\tau} \in (\frac{1-\theta}{\theta}, 1)$. Finally, at $\tau = \tilde{\tau}$, Lemma 4 implies that $\lambda_2 > -1$. As $\tau \downarrow \in \frac{1-\theta}{\theta}$, $A(\tau)B(\tau)^{-1} \to -\infty$ and Lemma 6.9 implies that $\lambda_2 < -1$. The existence of the desired value $\hat{\tau}$ then follows from the intermediate value theorem. ■

Lemma 6 nearly allows us to state an even stronger result. The lemma implies that the dynamical system defined by Eqs. (43) and (44) undergoes a flip bifurcation at $\tau = \hat{\tau}$. Thus there are solution sequences to those equations that display undamped oscillation. However, it is quite difficult to verify that such sequences will necessarily imply that the reserve requirement binds at each date, so we must remain agnostic as to whether undamped oscillation is a possibility.

6.3. *Discussion*

Our results regarding endogenously arising volatility have been obtained in a model where agents live only two periods and where capital depreciates completely after one period. These facts might lead one to interpret "a period" as being fairly long. Thus it is natural to ask whether the scope for endogenous fluctuations is likely to survive modifications of the environment that would allow us to interpret a period as a year or a quarter. The answer is that the use of a two-period model actually works strongly against finding endogenous volatility. This should arise under even weaker conditions in analogs of our economy where agents are longer-lived.

In order to see this, we draw on two results in the literature. First, Smith and Wang (1998) show that in economies with a finitely repeated costly state verification problem, the optimal contract will be a sequence of static contracts if verification costs are not too high. Second, Azariadis *et al.* (2000) demonstrate that overlapping generations models where agents are finitely lived, but live three or more periods, will necessarily generate endogenous volatility even in the absence of credit market frictions. Together these observations have the following implications. In a version of our economy where agents live, say, 55 periods, and where verification costs are not too high, the optimal contracts between borrowers and lenders will be exactly as we describe. And, endogenous volatility should emerge fairly easily.

It is also not conceptually difficult to allow only partial depreciation of capital. Doing so complicates the analysis, as it necessitates determining the replacement cost of capital. But this modification, too, is unlikely to weaken our results.

7. CONCLUDING REMARKS

We have analyzed the effects of central bank rediscounting, and expansionary monetary policy, in a small open economy with a domestic credit market friction. Under certain methods of financing government lending, we have seen that such a policy can potentially have a positive effect on long-run output levels only when the economy is in a development trap; otherwise it is counterproductive. We have also seen that such policies can give rise to endogenous volatility. On the other hand, central bank rediscounting on a large scale can render the high activity steady state determinate. In this sense, there is a tension in central bank rediscounting between the determinacy of equilibria—on the one hand—and the level of output and the volatility of output and inflation, on the other.

Of course these results have been obtained only for one particular—albeit important—kind of program: the central bank conducts monetary policy by rediscounting private loans. An important issue for future research concerns whether similar results can be obtained for other types of government programs—such as loans guarantees or direct lending—intended to inject credit or for other methods of conducting monetary policy.

APPENDIX

Proof of Proposition 6. As $\tau \uparrow \tau^* < \frac{1-\theta}{\theta}$, $k_2 \downarrow f^{-1}(q)$. Then, we have $D(k_2) \to (1-\delta)A(\tau^*)B(\tau^*)^{-1}$. Hence, for τ near (but below) τ^*, $D(k_2) < (>) 1$ holds iff $1 > (<)\ (1-\delta)A(\tau^*)B(\tau^*)^{-1}$. Using the definitions of $A(\tau)$, $B(\tau)$, and τ^*, this condition is equivalent to

$$[(1-\delta)R - 1]\left[R - (1-\theta) - \frac{\theta\psi}{f^{-1}(q)}\right]$$
$$> (<)[(1-\delta)R - (1-\theta)](R-1).$$

Rearranging terms gives condition (iv). ∎

Proof of Proposition 7.

(a) When $\tau = 0$, we have $A(0)B(0)^{-1} = \frac{R}{(1-\theta)}$, and—in addition—$\theta\psi H(k_2) = R - (1-\theta)$. Hence, when $\tau = 0$ holds,

$$D(k_2) = \left[\frac{(1-\delta)R}{1-\theta}\right]\left[\frac{R-(1-\theta)}{\theta\psi}\right]k_2.$$

Moreover, if credit is rationed ($\mu < 1$), $k_2 < \phi\alpha q$ holds. Hence if

$$\left[\frac{(1-\delta)R}{1-\theta}\right]\left[\frac{R-(1-\theta)}{\theta\psi}\right]\phi\alpha q \leq 1,$$

we have $D(k_2) < 1$, when $\tau = 0$.[26]

(b) We have

$$D(k_2) = (1-\delta)A(\tau)B(\tau)^{-1}k_2H(k_2) \geq (1-\delta)A(\tau)B(\tau)^{-1}f^{-1}(q)H(f^{-1}(q))$$
$$= (1-\delta)A(\tau)B(\tau)^{-1}.$$

[26] $k_2 < \phi\alpha q$ holds at $\tau = 0$ iff $\theta\psi H[\phi\alpha q] > R - (1-\theta)$.

Moreover, $A(0)B(0)^{-1} = \frac{R}{1-\theta}$. Hence if $\frac{(1-\delta)R}{(1-\theta)} > 1$, $D(k_2) > 1$ holds when $\tau = 0$. ■

Proof of Proposition 8.

(a) Condition (v) implies that, when $\tau = 0$, $D(k_2) < 1$ holds. If $\tau^* > \frac{1-\theta}{\theta}$ also holds, then as $\tau \uparrow \frac{1-\theta}{\theta}$, $A(\tau)B(\tau)^{-1} \uparrow \infty$. Since $D(k_2) = (1-\delta)A(\tau)B(\tau) \times k_2 H(k_2) \geq (1-\delta)\ A(\tau)B(\tau)^{-1}$ it follows that as $\tau \uparrow \frac{1-\theta}{\theta}$, $D(k_2) > 1$ must hold. Since $A(\tau)B(\tau)^{-1}$ and k_2 vary continuously with τ, for $\tau < \frac{1-\theta}{\theta}$, the existence of the desired value τ_c is implied by the intermediate value theorem.

(b) When $\tau = \tau_c$, $D(k_2) = 1$ holds. It follows that, at $\tau = \tau_c$, $T(k_2)^2 < 4D(k_2)$ is equivalent to $T(k_2) < 2$. But when $D(k_2) = 1$, Eq. (53) implies that

$$T(k_2) = 2 - \left[\frac{\theta\psi}{A(\tau_c)k_2}\right][k_2 H(k_2) - 1] < 2,$$

since for $\tau < \tau^*$, $k_2 H(k_2) > f^{-1}(q)H[f^{-1}(q)] = 1$. The claim then follows from continuity. ■

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